Air Data Calibration from Turning Flight

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Abstract

This paper discusses a method used to determine errors in true airspeed, angle of attack, and sideslip angle during turning flight. Once the true airspeed error is determined then other Pitot-static parameters are computed. This is an extension of a method programmed in the 1970s for use with turn data on the F-16. At the time it was assumed that the error in true airspeed was known, so we were solving for two components of the unknown wind. These were the north and east components. It was assumed that the vertical component of wind was zero.

In this present case, we are extending the method to three unknowns. Our measurements are ground speeds (North, East, and down) and an indicated true airspeed. The indicated true airspeed is computed from measurements of total pressure, static pressure and total temperature. The unknowns are two components of wind and an error in true airspeed. The method also assumes the vertical component of wind was zero. The fundamental equation is the vector airspeed equation. Once the wind components were computed, the true airspeed, angle of attack, and sideslip angle could be calculated.

Since the advent of the inertial navigation system (INS) in the 1970s, it has been possible to compute accurate values of air data parameters in dynamic maneuvers such as turns. However, this required the use of wind calibration runs conducted in wings-level 1-g flight where the air data system errors were known from conventional tests. In addition, INS data had small drift errors in the ground speeds. With the availability of the global positioning system (GPS) in the 1990s, an accurate value of ground speed was available. The mathematics and illustrating data for one such technique used in turning flight (that does not require the use of a wind calibration) will be presented in this paper.

A comparison is also made with a delta altitude method using GPS altitude.

Nomenclature

Note: speeds are in knots, angles are in degrees, and altitudes are in feet.

Abbreviations:

AFFTC = Air Force Flight Test Center

AIAA = American Institute of Aeronautics and Astronautics

EGI = embedded GPS/INS

ft = feet

GPS = global positioning system

INS = inertial navigation system

m = meter

Max = maximum (as in Max thrust)

Mil = Military (as in Mil thrust)

SFTE = Society of Flight Test Engineers

TPS = Test Pilot School

USAF = United States Air Force

Symbols:

f = a summation parameter in the x-direction

g = a summation parameter in the y-direction

h = a summation parameter involving true airspeed

g_0 = acceleration of gravity

P = ambient pressure (pounds/ft^2)

T = ambient temperature ($°K$)

δ = ambient pressure ratio

θ = ambient temperature ratio

α = angle of attack

α_{EGI} = angle of attack computed from EGI parameters

$\alpha_{A/C}$ = angle of attack from the aircraft system

H_{C_B} = balloon pressure altitude

ΔH = correction to be added to pressure altitude

Symbols (Concluded):

ΔV_t = correction to be added to true airspeed

D = down

E = east

γ = flightpath angle

A_y = flightpath lateral acceleration (ft/sec^2)

A_x = flightpath longitudinal acceleration (ft/sec^2)

A_z = flightpath normal acceleration (ft/sec^2)

$HGPS$ = GPS altitude

V_{gD} = ground speed down

V_{gE} = ground speed east

V_{gN} = ground speed north

$\vec{V}_g$ = ground speed vector

H_{C_i} = indicated pressure altitude

V_{ti} = indicated true airspeed

j = iteration number

K = Kelvin

V_{by} = lateral (y-body) axis airspeed

N_y = lateral load factor

V_{bx} = longitudinal (x-body) axis airspeed

N_x = longitudinal load factor

M = Mach number

N_z = normal load factor

N = north

∂ = partial derivative symbol

θ = pitch attitude

i = point number

H_C = pressure altitude

ϕ = roll attitude

β = sideslip angle

a = speed of sound

F^* = summation parameter to be minimized

$\sum$ = summation symbol

x = the x unknown = V_{wx}

y = the y unknown = V_{wy}

z = the z unknown = ΔV_t

P_t = total pressure (pounds/ft^2)

T_t = total temperature ($°K$)

V_t = true airspeed

V_{tD} = true airspeed down

V_{tE} = true airspeed east

V_{tN} = true airspeed north

$\vec{V}_t$ = true airspeed vector

ψ = true heading angle

V_{bz} = vertical (z-body) axis airspeed

V_{wD} = windspeed down

V_{wE} = windspeed east

V_{wN} = windspeed north

$\vec{V}_w$ = windspeed vector

Introduction

In the 1970s[1], 1980s[2], and 1990s[3] a number of aircraft tests projects have utilized inertial navigation system (INS) data in the reduction of aircraft performance and flying qualities data. (References 1, 2, and 3 are in Society of Flight Test Engineers (SFTE) or American Institute of Aeronautics and Astronautics (AIAA) symposiums or documents.) The INS gives you six parameters of interest for performance and flying qualities. These are three angles called Euler angles and three velocities in the north (N), east (E) and down (D) directions. The Euler angles are the heading from true north designated psi (ψ), the roll (or bank) angle designated phi (ϕ), and the pitch attitude designated theta (θ). The ground speed components from an INS are V_{gN}, V_{gE} and V_{gD}.

The INS method, in general, consisted of first performing a wind calibration or a series of wind calibrations as a function of altitude. These would be performed at speeds where an accurate airspeed and angle-of-attack calibration was previously obtained using conventional methods[4] such as stabilized pace, tower flyby's, and level radar tracked accelerations. During the wind calibration data points, one solves for wind components from the simple vector relationship that true airspeed equals ground speed plus windspeed. The full equations consist of three equations (one for each direction - N, E and D) with 12 parameters. The EGI yields six of those parameters ($V_{gN}, V_{gE,} V_{gD}, \phi, \theta$ and ψ). The aircraft Pitot-static system is used to compute true airspeed (V_t). The remaining five parameters are the three components of wind (V_{wN}, V_{wE} and V_{wD}) and angle of attack (α) and sideslip angle (β). So, one is left with (apparently) three equations and five unknowns. At the Air Force Flight Test Center (AFFTC), the usual assumptions used during the wind calibrations were that vertical wind (V_{wD}) and β were zero. That allows one to compute the horizontal components of wind. Others[2] have used a calibrated noseboom α to eliminate another unknown in the equations. Once the winds are known then for subsequent dynamic maneuvers α and β are computed and accelerations in the inertial axis can be transformed to the flightpath axis. Sounds good on paper, but for two decades we've been introducing errors in our data – albeit small, perhaps.

The problem is that we assumed we knew the ground speeds accurately. We didn't! The typical drift rate of an INS was on the order of 1 nautical mile per hour. Therefore, we had typical errors of about 1 knot in the horizontal ground speeds at any one time. Now (late 1990s) we have a new device designated as embedded GPS/INS (EGI). This combines the outputs of an INS with the velocities and position data from the GPS using a filter. The GPS specification accuracies for the horizontal speeds are 0.1 m/sec (0.19 knot). This small error does not drift with time. Therefore, we have introduced a new level of accuracy into our data. A number of papers[5,6] have been written on using GPS velocities to calibrate air data systems at 1 g. One paper[7] suggested what is perhaps a mathematical method similar to the one in this paper to derive air data at other than 1 g, but using EGI data strictly. These papers did not discuss the mathematics in detail or present any data.

Equation Development

$$\vec{V}_t = \vec{V}_g + \vec{V}_w \tag{1}$$

Solving for the magnitude of the true airspeed vector:

$$V_{ti} + \Delta V_t = \sqrt{\left[\left(V_{gN} + V_{wN} \right)^2 + \left(V_{gE} + V_{wE} \right)^2 + \left(V_{gD} + V_{wD} \right)^2 \right]} \tag{2}$$

We will assume the vertical wind is zero. Taking the square of both sides:

$$\left(V_{ti} + \Delta V_t \right)^2 = \left[\left(V_{gN} + V_{wN} \right)^2 + \left(V_{gE} + V_{wE} \right)^2 + V_{gD}{}^2 \right] \tag{3}$$

From here on in the derivation, we will simply strive to minimize the sum of the difference between the left and right side of the above equation. Defining a parameter we shall call F^* (F – star), we want to minimize the sum of this parameter simultaneously with respect to each of the three unknowns (V_{wN}, V_{wE}, ΔV_t). The iteration is the method of Taylor's series in three dimensions:

$$F^* = 0.5 \cdot \left(V_{tx}{}^2 + V_{ty}{}^2 + V_{tz}{}^2 - V_t^2 \right) \tag{4}$$

The 0.5 factor is just to eliminate ½ factors in the final formulation.

Where:

$$V_{tx} = V_{gN} + V_{wN} \tag{5}$$

$$V_{ty} = V_{gE} + V_{wE} \tag{6}$$

$$V_{tz} = V_{gD} \tag{7}$$

$$V_t = V_{ti} + \Delta V_t \tag{8}$$

Defining three more parameters: f, g and h:

$$f = \sum_{i=1}^{N} F_i^* \cdot V_{tx} \tag{9}$$

$$g = \sum_{i=1}^{N} F_i^* \cdot V_{ty} \tag{10}$$

$$h = \sum_{i=1}^{N} F_i^* \cdot V_t \tag{11}$$

There are N data points and N must be at least three. The x, y, z unknowns are as follows:

$$x = V_{wN}$$

$$y = V_{wE}$$

$$z = \Delta V_t$$

We will assume zero initial estimates for the unknowns.

$$x = y = z = 0$$

In addition, initialize f, g, h and the partial derivatives to zero.

$$f = g = h = 0$$

$$\partial f / \partial x = \partial f / \partial y = \partial f / \partial z = 0$$

$$\partial g / \partial x = \partial g / \partial y = \partial g / \partial z = 0$$

$$\partial h / \partial x = \partial h / \partial y = \partial h / \partial z = 0$$

Next we will generate a matrix of partial derivatives of f, g and h.

Summing from one to N:

$$\partial f / \partial x = \sum_{i=1}^{N} \left[\left(V_{tx} \right)^2 + F^* \right] \tag{12}$$

$$\partial f / \partial y = \sum_{i=1}^{N} \left[\left(V_{ty}(i) \right) \cdot \left(V_{tx}(i) \right) \right] \tag{13}$$

$$\partial f / \partial z = \sum_{i=1}^{N} \left[\left(-V_t(i) \right) \cdot \left(V_{tx}(i) \right) \right] \tag{14}$$

$$\partial g / \partial x = \sum_{i=1}^{N} \left[\left(V_{tx}(i) \right) \cdot \left(V_{ty}(i) \right) \right] \tag{15}$$

$$\partial g / \partial y = \sum_{i=1}^{N} \left[\left(V_{ty}(i) \right)^2 + F^* \right] \tag{16}$$

$$\partial g / \partial z = \sum_{i=1}^{N} \left[\left(-V_t(i) \right) \cdot \left(V_{ty}(i) \right) \right] \tag{17}$$

$$\partial h / \partial x = \sum_{i=1}^{N} \left[\left(V_{tx}(i) \right) \cdot \left(V_t(i) \right) \right] \tag{18}$$

$$\partial h / \partial y = \sum_{i=1}^{N} \left[\left(V_{ty}(i) \right) \cdot \left(V_t(i) \right) \right] \tag{19}$$

$$\partial h / \partial z = \sum_{i=1}^{N} \left[-\left(V_t(i) \right)^2 + F^* \right] \tag{20}$$

The following matrix formulation will solve for improved values for the unknowns.

$$\begin{Bmatrix} V_{wN} \\ V_{wE} \\ \Delta V_t \end{Bmatrix}_{j+1} = \begin{Bmatrix} V_{wN} \\ V_{wE} \\ \Delta V_t \end{Bmatrix}_{j} - \begin{bmatrix} \partial f / \partial x & \partial g / \partial x & \partial h / \partial x \\ \partial f / \partial y & \partial g / \partial y & \partial h / \partial y \\ \partial f / \partial z & \partial g / \partial z & \partial h / \partial z \end{bmatrix}^{-1} \cdot \begin{Bmatrix} f \\ g \\ -h \end{Bmatrix} \tag{21}$$

With improved values for the unknowns, simply return to the beginning of the algorithm and repeat the process until convergence occurs. This will usually occur after just a few steps. The parameter j is the iteration number. Once the winds are computed in the above process, one can proceed to compute angle of attack (α), sideslip angle (β) and the flightpath load factors (N_x, N_y, N_z).

Calculating α, β and V_t

The following matrices are used to transform the true airspeed from the flightpath axis (V_t) to the earth axis (V_{tN}, V_{tE}, and V_{tD}). The multiplication must be performed in the exact order of $\beta, \alpha, \phi, \theta, \psi$.

Heading (rotate about the z axis (or yaw)) [transform through ψ]:

$$[\psi] = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{22}$$

Pitch (rotate about y-axis) [transform through θ]:

$$[\theta] = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \tag{23}$$

Roll (rotate about x-axis) [transform through ϕ]:

$$[\phi] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \tag{24}$$

Angle of attack (transform through α):

$$[\alpha] = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \tag{25}$$

Sideslip angle (transform through β):

$$[\beta] = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{26}$$

The matrix summary form of the transformation from the flightpath axis true airspeed to the true airspeed in the earth axis (N, E, D) is as follows (where the matrix multiplication on the right side is performed from right to left):

$$\begin{Bmatrix} \left(V_{gN} + V_{wN}\right) \\ \left(V_{gE} + V_{wE}\right) \\ \left(V_{gD} + V_{wD}\right) \end{Bmatrix} = [\psi] \cdot [\theta] \cdot [\phi] \cdot [\alpha] \cdot [\beta] \cdot \begin{Bmatrix} V_t \\ 0 \\ 0 \end{Bmatrix} \tag{27}$$

From equation 27, we can solve for the winds:

$$\begin{Bmatrix} V_{wN} \\ V_{wE} \\ V_{wD} \end{Bmatrix} = [\psi]\cdot[\theta]\cdot[\phi]\cdot[\alpha]\cdot[\beta]\begin{Bmatrix} V_t \\ 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} V_{gN} \\ V_{gE} \\ V_{gD} \end{Bmatrix} \tag{28}$$

Equation 28 is the general matrix formula. During a typical wind calibration we will usually assume the vertical wind (V_{wD}) and β are equal to zero.

We now wish to perform the reverse transformation; that is to transform the components of true airspeed in the earth axis to the flightpath. To accomplish this is a matter of reversing the order of the matrix multiplication and take the transpose of each individual matrix. The following is the matrix formula:

$$[\beta]^T \cdot [\alpha]^T \cdot [\phi]^T \cdot [\theta]^T \cdot [\psi]^T \cdot \begin{Bmatrix} V_{tN} \\ V_{tE} \\ V_{tD} \end{Bmatrix} = \begin{Bmatrix} V_t \\ 0 \\ 0 \end{Bmatrix} \tag{29}$$

We can calculate all the velocities in equation 29 as follows:

$$V_{tN} = V_{gN} + V_{wN} \tag{30}$$

$$V_{tE} = V_{gE} + V_{wE} \tag{31}$$

$$V_{tD} = V_{gD} + V_{wD} \tag{32}$$

$$V_t = \sqrt{\left(V_{tN}{}^2 + V_{tE}{}^2 + V_{tD}{}^2\right)} \tag{33}$$

The airspeed components in the body axis (x, y, z) are calculated in the following matrix formula:

$$\begin{Bmatrix} V_{bx} \\ V_{by} \\ V_{bz} \end{Bmatrix} = [\phi]^T \cdot [\theta]^T \cdot [\psi]^T \cdot \begin{Bmatrix} V_{tN} \\ V_{tE} \\ V_{tD} \end{Bmatrix} \tag{34}$$

Next, one performs the transformation from the body axis to the flightpath axis through angle of attack and sideslip angle.

$$[\beta]^T \cdot [\alpha]^T \cdot \begin{Bmatrix} V_{bx} \\ V_{by} \\ V_{bz} \end{Bmatrix} = \begin{Bmatrix} V_t \\ 0 \\ 0 \end{Bmatrix} \tag{35}$$

Expanding the alpha and beta transpose matrices and writing them out:

$$\begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \cdot \begin{Bmatrix} V_{bx} \\ V_{by} \\ V_{bz} \end{Bmatrix} = \begin{Bmatrix} V_t \\ 0 \\ 0 \end{Bmatrix} \tag{36}$$

$$\begin{bmatrix} \cos\beta \cdot \cos\alpha & \sin\beta & \cos\beta \cdot \sin\beta \\ -\sin\beta \cdot \cos\alpha & \cos\beta & -\sin\beta \cdot \sin\alpha \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \cdot \begin{Bmatrix} V_{bx} \\ V_{by} \\ V_{bz} \end{Bmatrix} = \begin{Bmatrix} V_t \\ 0 \\ 0 \end{Bmatrix} \tag{37}$$

Multiplying out the above matrix and solving for angle of attack and sideslip yields the following:

$$\beta = \sin^{-1}\left(V_{by} / V_t \right) \tag{38}$$

$$\alpha = \tan^{-1}\left(V_{bz} / V_{bx} \right) \tag{39}$$

Indicated true airspeed (V_{ti}) is computed from total air pressure (P_t), static pressure (P_s), and total temperature (T_t).

Turn Data

To illustrate, the method was applied to turn data from an F-15 at 0.85 Mach number and 30,000 feet. Figure 1 is a time history of a maximum thrust turn. Indicated true airspeed shown is computed assuming zero error (commonly called ‘position error’). The plot shows the north, east, and down inertial speeds. These speeds were from the EGI unit. The specification velocity accuracy of such a device is 0.10 m/sec (0.19 knot). Figure 2 is true airspeed versus time after corrections were applied to indicated true airspeed and to the inertial speeds. True airspeed can be computed two ways: from the Pitot-static indicated true airspeed plus the derived correction, and from inertial speeds plus winds. The derived values for the airspeed correction and the winds are as follows:

$\Delta V_t = -5.67 \text{ knots}$ (correction to be added)

$V_{wN} = -9.61 \text{ knots}$

$V_{wE} = -74.23 \text{ knots}$ {or $V_w = 74.85 \text{ knots}$ @ 262.6° from true north}

This method assumed that these three unknowns were constant. We realize they cannot be exactly constant in the real atmosphere; however, the derived numbers are those that produce the minimum least squares error between airspeed computed from the two different ways discussed above.

There were also three 1-g wind calibrations performed on the same flight, at the same pressure altitude, and within a 10-minute period of the maximum thrust turn. Table 1 presents those three data points along with the maximum thrust turn point and an additional military thrust turn.

Table 1. Wind Calibration Summary

Event	Time (hours/minutes)	Windspeed (knots)	Wind Direction (degrees)
Wind calibration	17:44	72.5	266.0
Wind calibration	17:49	75.8	261.8
Wind calibration	17:53	75.2	259.8
Military thrust turn	17:47	75.3	263.8
Maximum thrust turn	17:50	74.8	262.6

In Figure 1 you will find indicated true airspeed, and the north (x), east (y), and down (z) components of inertial speed. The down component has been multiplied by a factor of 10 to make it more visible on the plot, since this was a sustained constant altitude turn with small vertical velocity variations.

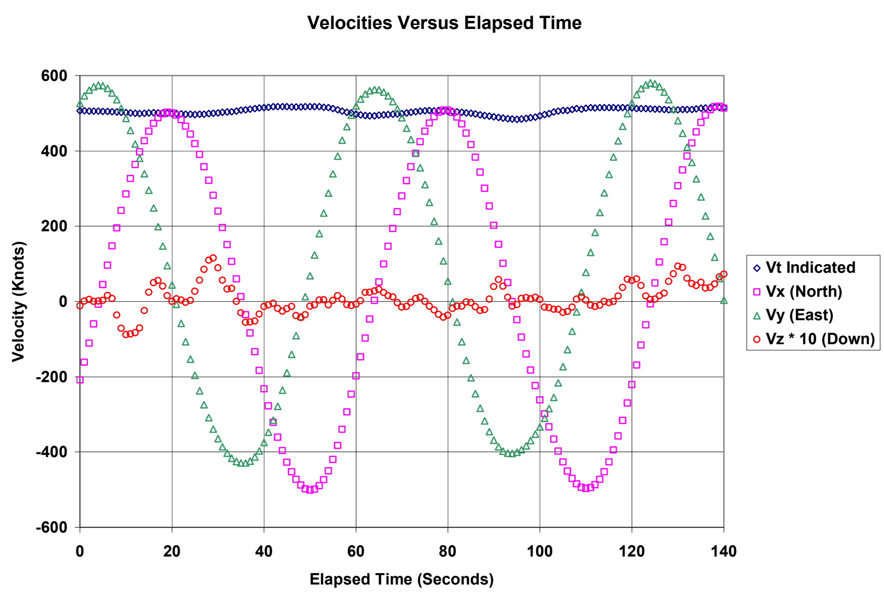

Figure 1. Turn Data Time History

Figure 2 compares true airspeed computed two different ways. The first way is from indicated Pitot-static plus the correction. The second method is from inertial speeds plus winds. The formulas for these are as follows:

Pitot-static plus correction: $V_t = V_{ti} + \Delta V_t$ (40)

Inertial plus winds: $V_t = \sqrt{\left(V_{gN} + V_{wN}\right)^2 + \left(V_{gE} + V_{wE}\right)^2 + V_{gD}{}^2}$ (41)

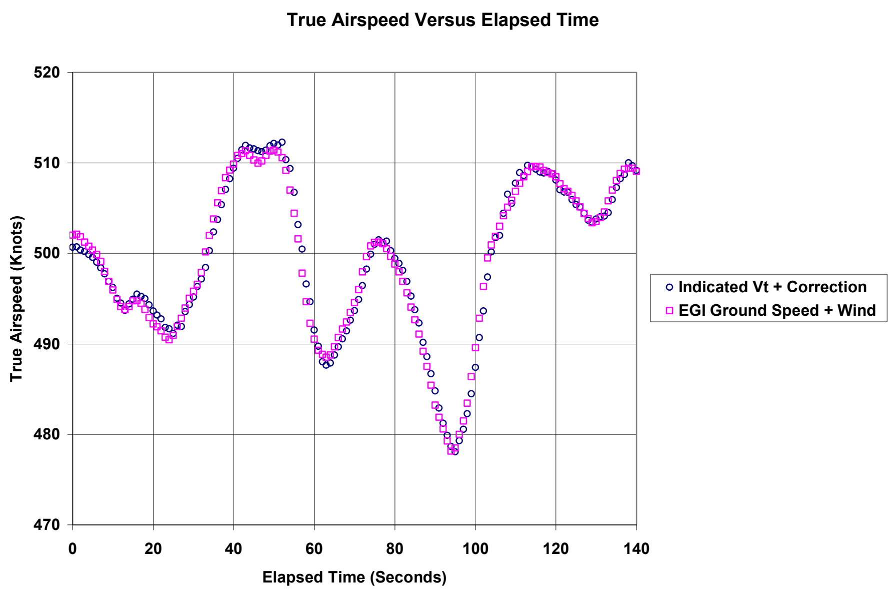

Figure 2. True Airspeed versus Time

The difference between the two parameters is shown in Figure 3.

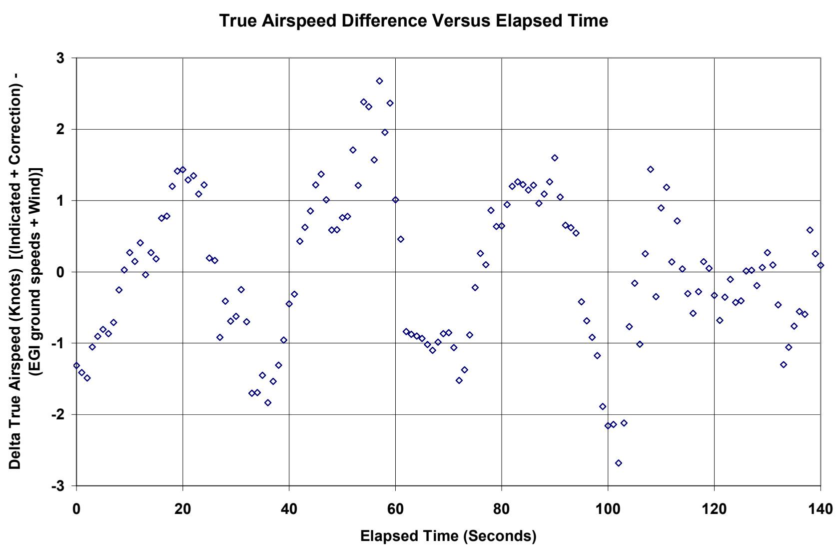

Figure 3. Airspeed Difference versus Time

The mean difference above (Figure 3) is zero. The standard deviation of the velocity error is 1.06 knots. What is the significance of the value of ΔV_t? First, the average value of indicated true airspeed was 505.0 knots and with a correction to be added of –5.67 knots that comes to just over 1 percent. For this turn at about three g's, this 1-percent error in true airspeed compares to near zero error at the same altitude and Mach number, but at 1 g.

Angle-of-Attack Variation

Next, using the transformation formulas presented earlier, the angle of attack can be computed now that we have the winds along with the EGI velocities and angles. The aircraft's air data system α can be plotted versus the EGI derived α to develop a correction for the aircraft's system. This data is presented in Figure 4.

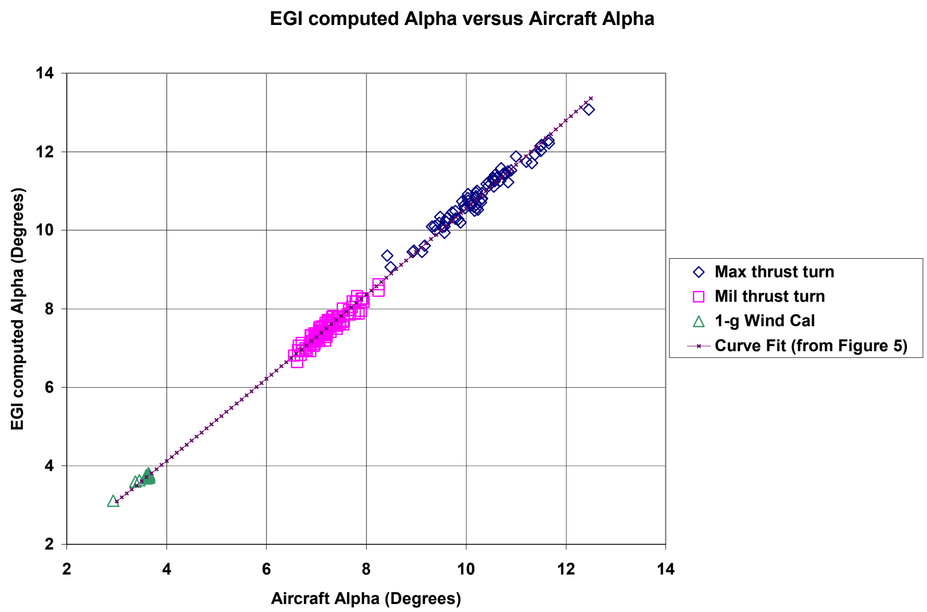

Figure 4. Angle-of-Attack Comparison

The difference between the aircraft and EGI Alpha is plotted in Figure 5. The fairing for the data in Figure 4 was derived from the curve fit in Figure 5. The statistics of the curve fit are shown in Table 2.

Table 2. Angle-of-Attack Statistics

Data Source	Load Factor (N_Z)	Mean Deviation (degrees)	Standard Deviation (degrees)
Maximum Turn	3	-0.034	0.176
Military Turn	2	0.027	0.126
Wind Cals	1	-0.022	0.049
All Data	1 to 3	0	0.138

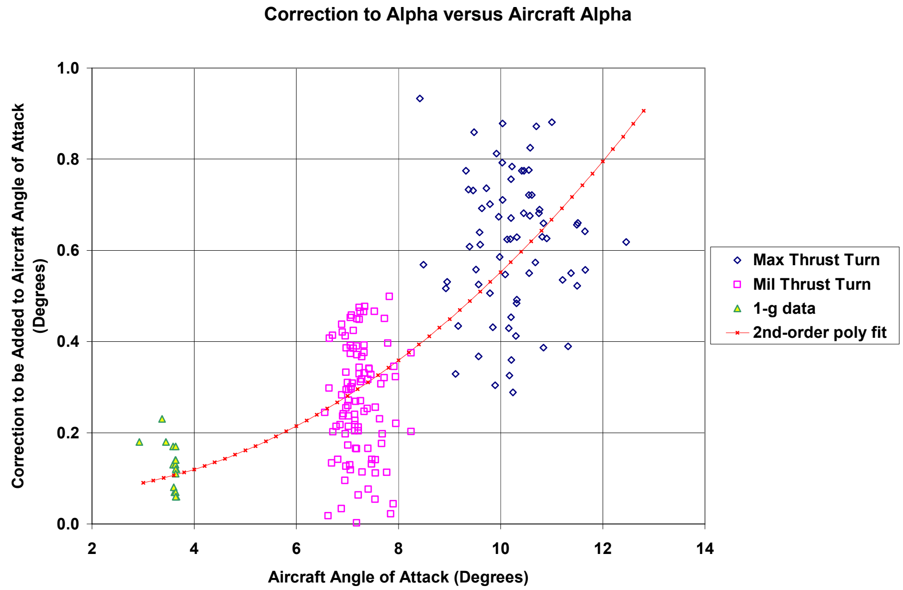

Figure 5. Angle-of-Attack Correction To Be Added

Table 3 summarizes the computed position error data from these turns and from 1-g data. The 1-g data is simply the curve value that was applied to all data, independent of angle of attack. The Mach number and altitude errors were computed as if all of the errors in the Pitot-static system were from static pressure.

Table 3. Air Data Calibration Data Summary

N_z	M_i	H_{C_i}	ΔV_t	ΔM_i	ΔH	α_{EGI}	$\alpha_{A/C}$
(g)		(feet)	(knots)		(feet)	(degrees)	(degrees)
3.0	0.8591	30,429	-5.665	-0.0110	-251.	10.84	10.23
2.0	0.8544	30,289.	-2.526	-0.0049	-112.	7.51	7.24
1.0	0.86	30,000.	+1.30	+0.0022	+52.	3.70	3.40

As seen in Figures 4 and 5, there was a variation in angle of attack during each of the turns. If we compute the deviation in true airspeed versus angle of attack, one can see an effect of angle of attack. The deviation in true airspeed is the difference between true airspeed computed from ground speed plus winds and from indicated true airspeed. The equation for that deviation is as follows.

$$\Delta V_t = \sqrt{\left[\left(V_{gN} + V_{wN}\right)^2 + \left(V_{gE} + V_{wE}\right)^2 + V_{gD}{}^2\right]} - \left(V_{ti}\right) \quad (42)$$

For each turn, the parameters ΔV_t, V_{wN} and V_{wE} were determined in such a manner that the sum of $\left(\Delta V_t\right)$ for the N data points was exactly zero. However, the plot of ΔV_t versus time (Figure 3) illustrates a variation. We suspect that error may be a function of angle of attack. Figure 6 is the plot of equation 42 versus EGI computed angle of attack.

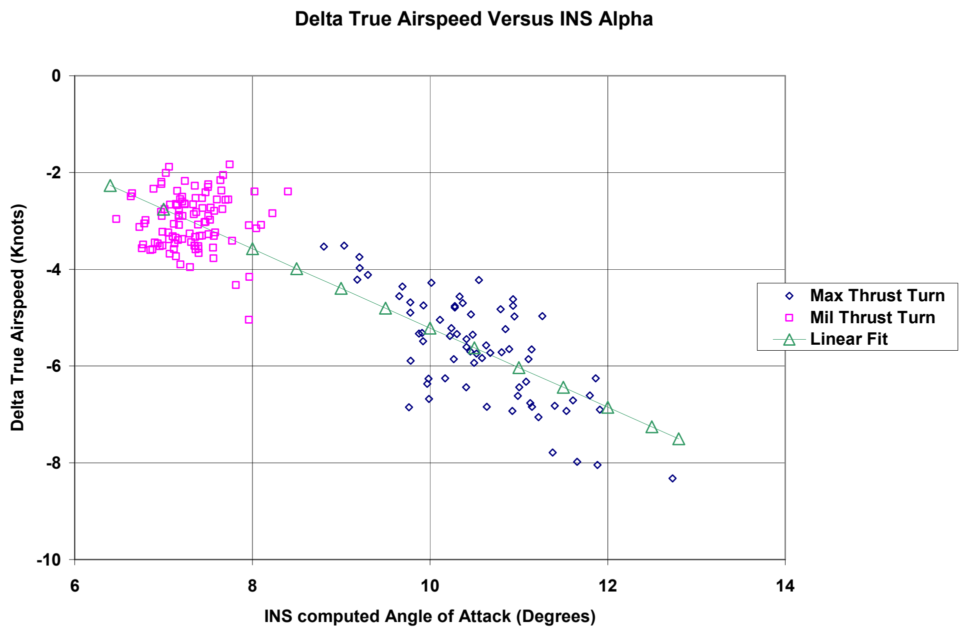

Figure 6. True Airspeed Error versus Angle of Attack

The 1-sigma error in ΔV_t from the above linear fit is 0.68 knots. For the Max thrust turn alone, the error is 0.76 knots. This compares to the 1.06-knot standard deviation of the time history in Figure 3. The linear fit reduces the error by 28 percent.

There is an additional source of information—the GPS altitude. We start with a known position error for the aircraft pressure altitude at 1 g. This position error would have been determined from conventional airspeed calibration maneuvers, including the tower flyby, pace (Reference 4), and 'cloverleaf maneuver' (Reference 8). First, we need to determine a relationship between geometric altitude (GPS) and pressure altitude. Although we will use only aircraft data for this, a comparison with weather balloon data will be shown.

The following chart (Figure 7) shows aircraft data along with a comparison with weather balloon data from near the time of the data. The balloon data is shown for comparison purposes only. However, it should be noted that it is good practice to make comparisons with the balloon data as a quality check on your aircraft data.

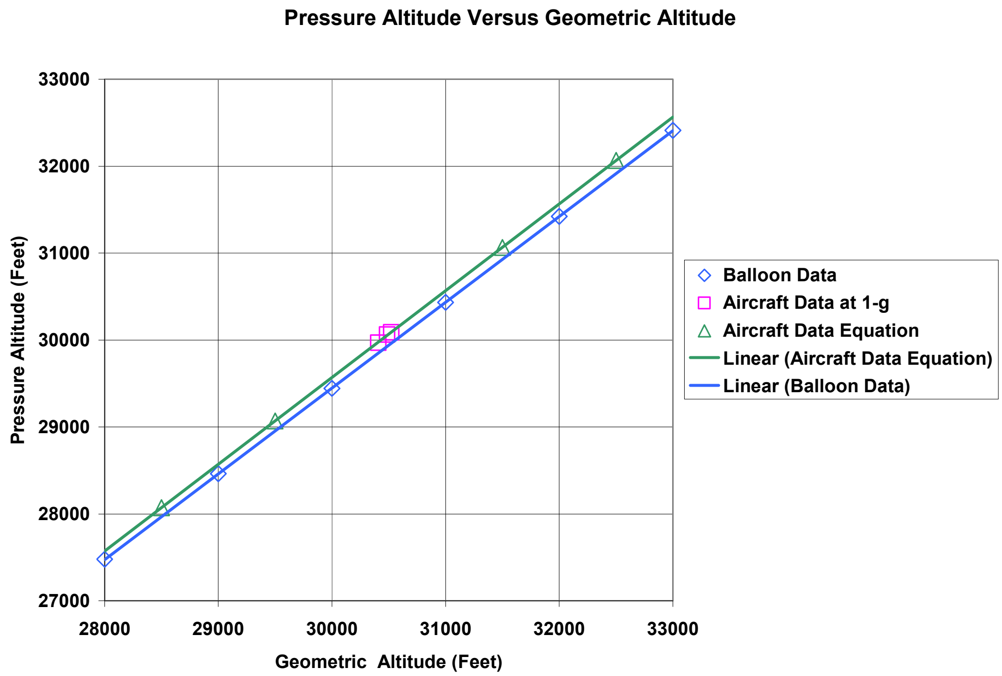

Figure 7. Comparison of Aircraft and Weather Balloon Altitude Data

The three aircraft data points, which were before and after the Mil and Max thrust turns are summarized in Table 4. The balloon data points are from the straight line curve fit in the plot. The equation for the aircraft pressure altitude is from the following.

$$T_{as} = 288.15 - (1.9812/1000) \cdot H_C \tag{43}$$

$$H_C = \left(T_{as} / T \right) \cdot HGPS + C \tag{44}$$

$$C = H_C - \left(T_{as} / T \right) \cdot HGPS \tag{45}$$

H_C - Pressure altitude (feet)

T_{as} - Standard day ambient temperature (deg K)

T - Test day ambient temperature (deg K)

Table 4. GPS versus Pressure Altitude Comparison

$HGPS$ (feet)	H_C (feet)	H_C Fit (feet)	T (deg K)	T_{as} (deg K)	$\left(T_{as}/T\right)$	C	H_{C_B} (feet)	ΔH_1 (feet)	ΔH_2 (feet)
30,487	30,062	30,054	228.8	228.6	0.9991	-397	29,928	126	-8
30,408	29,969	29,975	229.1	228.8	0.9986	-396	29,850	125	6
30,522	30,087	30,089	229.0	228.5	0.9980	-374	29,963	126	2
30,477	30,049	30,039	229.0	228.6	0.9986	-389	29,914	126	0

Note: Last row contains average values.

H_{C_B} - Balloon pressure altitude

H_C Fit – From equation (44).

$$\Delta H_1 = H_C \text{ Fit} - H_{C_B} \tag{46}$$

$$\Delta H_2 = H_C \text{ Fit} - H_C \tag{47}$$

Altitude Error

Now we have two methods to compute the so-called 'position error'- delta H and delta V method. We will assume that all of the error in altitude is due to static pressure. That is, there is zero total pressure and total temperature error. The first method we will call the delta H method.

Delta H Method

The delta H method is simply taking the difference between a pressure altitude computed from the GPS altitude minus the indicted pressure altitude. For the case of this data set, the 'indicated' pressure altitude (H_{C_i}) already had a 1-g calibration applied to it.

$$\Delta H = \left(\left(T_{as}/T\right) \cdot HGPS + C\right) - H_{C_i} \tag{48}$$

Delta V Method

The delta V method takes the results of the correction to true airspeed discussed earlier and computes a corrected pressure altitude. From the true airspeed (V_t) and total temperature (T_t), we can compute Mach number(M). An iteration is required; use the indicated M for the first iteration.

$$T = \frac{T_t}{\left[1 + 0.2 \cdot M^2\right]} \quad (49)$$

$$\theta = \frac{T}{288.15} \quad (50)$$

$$M = \frac{V_t}{\left(661.48 \cdot \sqrt{\theta}\right)} \quad (51)$$

From Mach number and total pressure (P_t), compute ambient pressure (P):

$$P = \frac{P_t}{\left(1 + 0.2 \cdot M^2\right)^{3.5}} \quad \text{formula valid for } M < 1 \quad (52)$$

From P , compute the pressure altitude (H_C):

$$\delta = \frac{P}{2116.22} \quad \text{ambient pressure ratio} \quad (53)$$

$$H_C = \frac{\left[1 - \delta^{(1/5.2559)}\right]}{6.87559E - 6} \quad \text{valid for } H_C < 36{,}089 \text{ feet} \quad (54)$$

Just as for the delta H method the correction is just the corrected value in equation (54) minus the indicated value:

$$\Delta H = H_C - H_{C_i} \quad (55)$$

The two methods are compared in Figure 8. All of the data points less than an angle of attack of nine degrees are from the Mil thrust turn; the ones above are from the Max thrust turn. The curve fit is of the combined data set - both the delta H and delta V method.

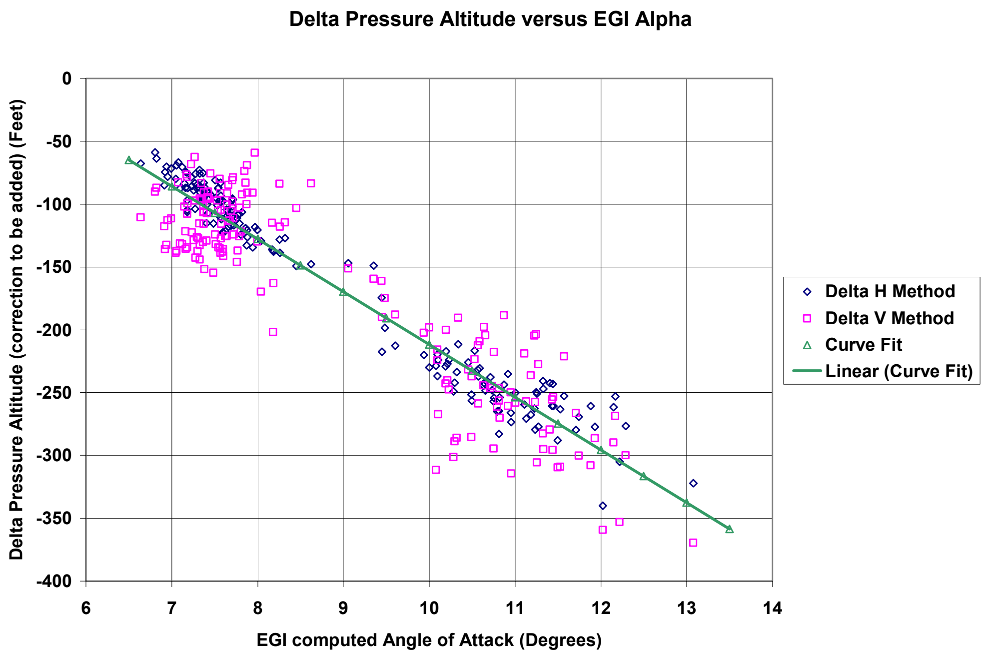

Figure 8. Delta H versus Angle of Attack

Table 5 presents the statistics of the above data. The first row for each data set is the mean value and the second row is the one sigma variation.

Table 5. Altitude Error versus Angle of Attack

Data Source	α_{EGI} (degrees)	ΔH (feet)	ΔH Deviation From Linear Fit (feet)
Max Thrust Turn Delta H Method	10.84 0.78	-247 31	0 19
Max Thrust Turn Delta V Method	10.84 0.78	-251 47	4 34
Mil Thrust Turn Delta H Method	7.51 0.37	-100 20	10 -7
Mil Thrust Turn Delta V Method	7.51 0.37	-112 25	29 5

From the above tabulated numbers, one can make a few observations. First, the 'scatter' in the data, as measured by the variation about the linear fit, is remarkably small. We feel this is due to the extremely accurate GPS velocities and altitudes and to the high resolution of the aircraft Pitot-static system data. There is a significant variation of altitude error with angle of attack; the slope of the curve is 42 feet per degree of angle of attack. Second, there is essentially no difference in magnitude between the delta H and delta V results. This suggests that all of the error in altitude is due to static source error. This is because the delta H method is a direct measure of altitude error and therefore, must be a direct measure of static source error. If there was a difference between the delta H and delta V results, then that difference could be attributable to either a total pressure or total temperature error as a function of angle of attack.

Summary

A method was presented for calibrating air data systems at angles of attack larger than those normally obtained at 1 g using data obtained during turning flight.

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